MATH 16A MIDTERM 1(PRACTICE 1)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.
CALCULATORS ARE NOT PERMITTED
YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK
SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE
REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT
THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Determine the domains of the following functions:

(a)

$$\ln(x^2+1)$$

Solution:

Domain of $\ln(x)$ is $(0, \infty)$

$x^2+1 > 0$ for all x

$\Rightarrow$ Domain of $\ln(x^2+1)$ is all real numbers

(b)

$$\sqrt{\frac{x+2}{x-1}}$$

Solution:

$\frac{x+2}{x-1} \geq 0 \Rightarrow$

$x+2 \geq 0$, $x-1 > 0$ $\Downarrow$ $x \geq -2$, $x > 1$ $\Downarrow$ $x > 1$

OR

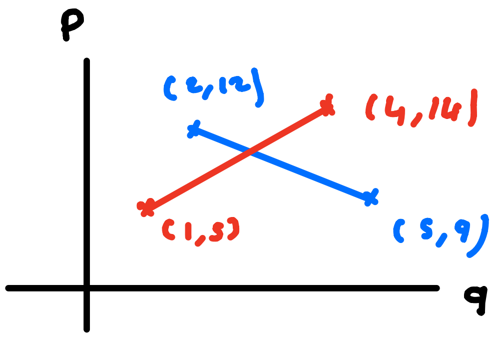
$x+2 \leq 0$, $x-1 < 0$ $\Downarrow$ $x \leq -2$, $x < 1$ $\Downarrow$ $x \leq -2$

$\Rightarrow$ Domain $= (-\infty, -2] \cup (1, \infty)$

2. (25 points) A product is to be supplied and sold. If the price per unit is 5 dollars the supplier is willing to provide 1 unit. If the price per unit is 14 dollars the supplier is willing to provide 4 units. If the price per unit is 12 dollars the demand is 2 units. If the price per unit is 9 dollars the demand is 5 units.

   (a) Determine the supply and demand equations in this situation.

   Solution:



Supply : $p - 5 = \frac{14-5}{4-1}(q-1)$

$\Rightarrow p - 5 = 3(q-1)$

$\Rightarrow p = 3q + 2$

Demand : $p - 9 = \frac{9-12}{5-2}(q-5)$

$\Rightarrow p - 9 = -(q-5)$

$\Rightarrow p = -q + 14$

   (b) For what prices per unit will there be a surplus?

   Solution:

Equilibrium : $3q + 2 = -q + 14$

$\Rightarrow 4q = 12 \Rightarrow q = 3$

$\Rightarrow p = 11$ dollars

If the price is <u>more</u> than 11 dollars there will be a surplus

3. Calculate the following limits. If they do not exist determine if they are ∞ or $-\infty$.

(a)

$$\lim_{x\to 1} \frac{\ln(x+1)}{x+1}$$

Solution:

$$\lim_{x\to 1} \frac{\ln(x+1)}{x+1} = \frac{\ln(1+1)}{1+1} = \frac{\ln(2)}{2}$$

(b)

$$\lim_{x\to\infty} (\ln(2x+1) - \ln(3x-2))$$

Solution:

degree 1

$$\lim_{x\to\infty} (\ln(2x+1) - \ln(3x-2)) = \lim_{x\to\infty} \ln\left(\frac{2x+1}{3x-2}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{2x+1}{3x-2}\right) = \ln\left(\frac{2}{3}\right)$$

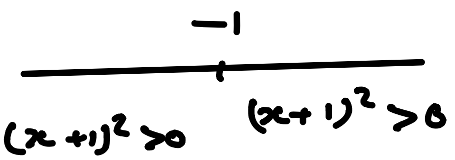
(c)

$$\lim_{x\to -1} \frac{\sqrt{1-x}}{x^2+2x+1}$$

Solution:

$$\lim_{x\to -1} \sqrt{1-x} = \sqrt{1-(-1)} = \sqrt{2} > 0$$

$$\lim_{x\to -1} x^2+2x+1 = \lim_{x\to -1} (x+1)^2 = 0^+$$



$$\Rightarrow \lim_{x\to -1} \frac{\sqrt{1-x}}{x^2+2x+1} = \infty$$

(DNE)

4. Using limits, calculate the derivative of $f(x) = 3x^{-2}$. $= \frac{3}{x^2}$

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h} = \lim_{h \to 0} \frac{3x^2 - 3(x+h)^2}{h(x+h)^2x^2}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} - \cancel{3x^2} - 6xh - 3h^2}{h(x+h)^2x^2}$$

$$= \lim_{h \to 0} \frac{-6x - 3h}{(x+h)^2x^2} = \frac{-6x}{(x+0)^2x^2} = \frac{-6}{x^3}$$

5. let $f(x) = \frac{3x^2+2x+a}{x^2-2x+1}$, for a a real number. Is there a value of a for which $\lim_{x\to 1} f(x)$ exists? Carefully justify you answer.

Solution:

$$\lim_{x \to 1} 3x^2 + 2x + a = 5 + a$$

$$\lim_{x \to 1} x^2 - 2x + 1 = \lim_{x \to 1} (x-1)^2 = 0$$

$\Rightarrow$ If $5 + a \neq 0$ limit of quotient DNE.

Assume $5 + a = 0 \Rightarrow a = -5$

$$3x^2 + 2x - 5 = (3x+5)(x-1)$$

$$\Rightarrow \frac{3x^2+2x-5}{x^2-2x+1} = \frac{3x+5}{x-1}$$

$$\lim_{x \to 1} 3x + 5 = 8 \neq 0$$

$$\lim_{x \to 1} x - 1 = 0$$

$$\Rightarrow \lim_{x \to 1} \frac{3x+5}{x-1} \text{ DNE}$$

$\Rightarrow$ There is no value of a we can choose to make the limit exist.

END OF EXAM